Journal of Atmospheric and Solar-Terrestrial Physics 145 (2016) 34–44

Contents lists available at ScienceDirect

Journal of Atmospheric and Solar-Terrestrial Physics

journal homepage: www.elsevier.com/locate/jastp

Multifractal analysis of lightning channel for different categories of lightning

F.J. Miranda [a,*], S.R. Sharma [b]

[a] Department of Mathematics and Statistics, Federal University of the Valleys of Jequitinhonha and Mucuri, Diamantina, Brazil
[b] Department of Physics, Amrit Science College, Tribhuvan University, Kathmandu, Nepal

ARTICLE INFO

Article history:
Received 28 August 2015
Received in revised form
30 March 2016
Accepted 31 March 2016
Available online 2 April 2016

Keywords:
Lightning channel
Multifractal spectrum
Fractal dimension

ABSTRACT

A study from the point of view of complex systems is done for lightning occurred at Diamantina, Sete Lagoas and São José dos Campos, during the summer from September 2009 to April 2010. For the first time, multifractal analyses were performed for different lightning categories: two-dimensional, three-dimensional, non-branched, branched, cloud, cloud-to-ground, single and multiple. We found that when using two-dimensional images of natural lightning embedded in three dimensions to perform multifractal analysis, the interpretation of the multifractal spectrum must be restricted to identification of the multi (mono) fractal character of lightning channel and to estimation of fractal dimension. We have also observed that, on the average, each category has a specific value of fractal dimension. Categories in which branches and tortuosity are more usual, like branched and cloud categories, exhibited largest fractal dimensions due to more complexity of lightning channels. The results suggest that single and multiple lightning have similar complexities in their channels, leading to the same average values of fractal, information and correlation dimensions for both categories.

1. Introduction

Measurements of Lightning electromagnetic radiation have been extensively carried out and have revealed many features of lightning in time domain. For example, from lightning electric field waveform measurements, the average time between K-changes was observed in the range of 8–20 ms (Kitagawa and Brook, 1960; Miranda et al., 2003; Thottappillil et al., 1990). Measurements have also been made in frequency domain using narrow band filters (Horner and Bradley, 1964; Le Vine, 1987; Schafer and Goodall, 1939). Similarly, frequency spectra were obtained by measuring the signal in a wide band measurement system in time domain and Fourier transforming it. This procedure in studies of lightning radiation has widely been used after Serhan et al. (1980). Lightning electric field waveform measurements has also been associated with wavelet analyses to obtain temporal information about lightning spectrum (Esa et al., 2014; Li et al., 2013; Miranda, 2008; Sharma et al., 2011). Measurements of lightning radiation can also be used to verify bifurcation of lightning channel in space (Willett et al., 1995). However, although the powerful procedure of measuring electromagnetic radiation of lightning in time domain can reveal some information of lightning channel behavior in space, the use of images (still pictures, streak photography or videos) have revealed to be a more suitable procedure for observation of lightning behavior in space. Historically, the first observations of lightning images were done by Schonland et al. (1935) and Schonland et al. (1938), and from them other studies have been done (Antunes et al., 2015; Flache et al., 2008; Idone and Orville, 1988; Idone, 1995; Saba et al., 2004; Valine and Krider, 2002; Willett et al., 1995; Winn et al., 1973). Although these studies so far mentioned, are observations of lightning channel geometry (tortuosity or bifurcations), they do not address the fractal or multifractal nature of lightning channel.

It is known that, besides the intensification of the content of high frequency in the electromagnetic spectrum and more jagged radiation fields (Le Vine and Meneghini, 1978), the increase of tortuosity of lightning channel also makes lightning behave as fractal antenna (Valdivia et al., 1997, 1998; Vecchi et al., 1994). Models of fractal dynamics and radiation of intracloud micro-discharges and of sprites have been done by Iudin et al. (2003), Hayakawa et al. (2007) and Hayakawa et al. (2008). Some researchers have seen that radiation fields emitted by lightning are characterized by the same fractal dimension of lightning channel (Gou et al., 2009; Vecchi et al., 1994). However, Lupò et al. (2000) obtained results in which dissimilarities among fractal dimensions of channel and radiation fields were found. There are a lot of studies in the literature, concerning the understanding the fractal nature of lightning. They can range from simulation of electrical

* Corresponding author.
E-mail address: fjm_01@hotmail.com (F.J. Miranda).

http://dx.doi.org/10.1016/j.jastp.2016.03.012
1364-6826/

discharges in dielectric, computational simulation and modelling of lightning channel, measurements of radiation fields up to fractal analysis of still pictures. The results of fractal dimension from this diversity of techniques can be seen in the literature in a range between 1.1 and 1.9 (Amarasinghe and Sonnadara, 2008; Kawasaki and Matsuura, 2000; Lupò et al., 2000; Tsonis, 1991).

The values of fractal dimensions presented by Amarasinghe and Sonnadara (2008) indicate that simulated electrical discharges and surface discharges show fractal dimensions slightly larger than natural lightning. For example, simulation of branching discharges gas by Niemeyer et al. (1984) naturally lead to structures with fractal dimension equals 1.75 ± 0.02. In turn, Tsonis (1991) analyzed a set of lightning photographs and found an average fractal dimension equal to 1.34 ± 0.05.

Most studies on the fractal nature of lightning channels are made from two-dimensional simulation or two-dimensional images. However, computer simulations show that the fractal dimension of a three-dimensional lightning is about 10–13% higher than the fractal dimension of its projection on a plane (Amarasinghe and Sonnadara, 2008; Sañudo et al., 1995). More recently Perera and Sonnadara (2013) made two-dimensional and three-dimensional simulations of electrical discharges. In these simulations they used an improved *Dielectric Breakdown model* (DB model) in which the exponent (η) parameterizes the relationship between the local electric field and the probability of growth of the discharge pattern. They observed "bush" type discharges and "branched" type discharges, depending on the power (η) of the local electric field. According to them, discharge patterns similar to actual lightning were obtained when the exponent η was equal to about 5.2.

According to Lupò et al. (2000), the works developed previously to their work considered only the influence of tortuosity of lightning channels in the results. Thus, they are the first to consider the influence of the branches in the results.

Other works have plunged deeper into the complex nature of lightning. Gou et al. (2007, 2009) found that the return stroke electric signals exhibit strong degree of multifractality and singularity, the multifractal spectrum fits to the modified version of binomial multifractal model and that multifractal spectrum can be regarded as "*fingerprints*" of return strokes. Also, according to Gou et al. (2009), the fractal dimensions of the electric signals of radiation ranged from 1.2 to 1.5 with an average of 1.3 and the peak current of a return stroke has been related to the electric charge deposited in the leader channel by the fractal dimension obtained.

To the best of our knowledge, no study of multifractality of lightning has been done for different categories of lightning. So, as a general objective in this work, we intend to analyze multifractal nature of different categories of lightning. These categories are groups formed by lightning selected according to some specific behavior in space.

For this study, lightning images made by a normal camera were used and eight categories were analyzed.

More specifically, the objectives of this study are:

a) Perform and compare multifractal analysis of two-dimensional natural lightning and three-dimensional simulated lightning;
b) Perform and compare multifractal analysis of non-branched ground lightning (Fig. 1a) and branched ground lightning (Fig. 1b) categories. For both categories of lightning, the term "ground" refers to a cloud-to-ground or ground-to-cloud

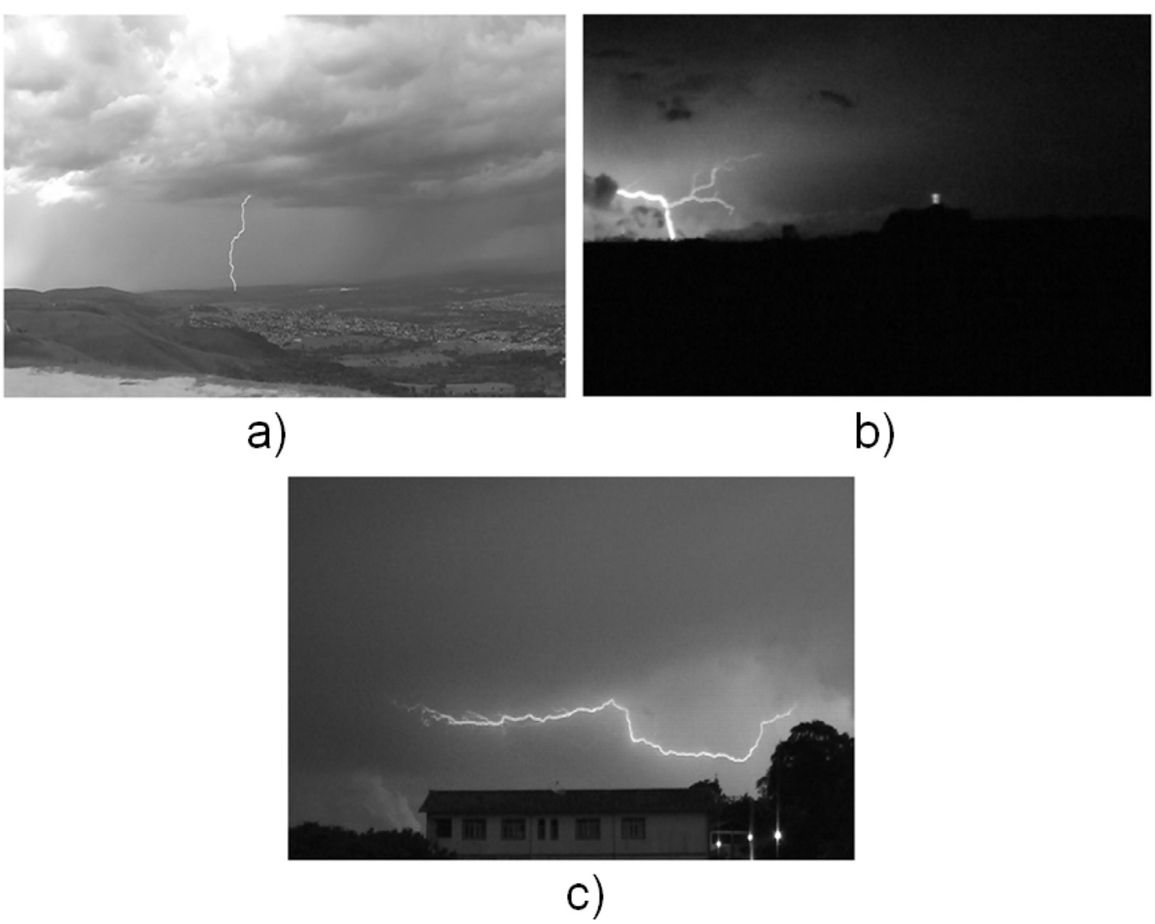

Fig. 1. Exemplification of lightning used in the composition of the categories studied in this work: a) Non-branched ground lightning; b) Branched ground lightning; c) Cloud lightning.

lightning connecting ground and cloud. If it is a multiple lightning, only lightning with a unique shape of channel at all detected return strokes were selected for analysis. The term "branched" indicates presence of branches while the term "non-branched" indicates absence of branches;

c) Perform and compare multifractal analysis of ground lightning (Fig. 1a and b) and cloud lightning categories (Fig. 1c). Ground lightning category included non-branched, branched, single and multiple lightning which connected ground and cloud. Also, in case of multiple lightning, only lightning with a unique shape of channel at all detected return strokes were selected for analysis. Cloud lightning category included any lightning with a unique shape of channel which started inside of the cloud and did not touch the ground;

d) Perform and compare multifractal analysis of single and multiple lightning categories. Single lightning category consisted of ground lightning with only one return stroke. Multiple lightning category consisted of ground lightning with two or more return strokes and in this case, only lightning with a unique shape of channel at all detected return strokes were selected for analysis.

We believe that the main importance of this work lies in its contribution to the better understanding of the complexity and the evolution of the structure of channels of lightning.

2. Methodology

The development methodology of this paper is as follows: in the first stage, data were acquired in campaigns during summer thunderstorms in three cities of Brazil. Still in this stage, the data were also prepared to serve as input for calculations. In the second stage, multifractal spectra of the data obtained in the first stage have been calculated, generating results which were analyzed in the third stage.

In this work, two-dimensional natural lightning, or simply, two-dimensional lightning refers to images of natural lightning. Three-dimensional simulated lightning refers to a lightning in three dimensions, generated through a computational simulation.

Below, details of the first and second stages are shown. The results analyzed in the third stage are shown in Section 3. Finally, conclusions are presented in Section 4.

2.1. Data acquisition

Lightning images were used to compute the multifractal spectrum of lightning channel. All images were obtained with a 1/6" CCD camera (VDR300 Panasonic) with 3 CCD sensors, with 800k pixels in each sensor. Lightning were filmed during the summer from September 2009 to April 2010, over the following Brazilian cities: São José dos Campos (23°11′S and 45°53′W), Sete Lagoas (19°27′S and 44°16′W) and Diamantina (18°14′S and 43°36′W). Only movie containing channels with good visibility were selected for analysis of lightning in space. When a ground or a cloud lightning occurred in the movie, a sequence of frames including the observed lightning and corresponding to about 2 (two) seconds was analyzed in order to investigate some possible variation in the characteristics of the channel. Due to the fact that the preliminary discharges (breakdown and leader) were less intense and usually invisible to the naked eye and according to the common sense in the literature that the lightning channel is the path of the return stroke or main discharge, we defined the main channel of the first return stroke in a ground lightning or the channel of the main discharge in a cloud lightning, as the lightning channel and determined the category of lightning. For most of lightning analyzed in this work, the main channel was clearly larger and stronger in luminosity than the branches and also connected to ground in the case of ground lightning, making easy the identification of the main channel and branches. Then, for all lightning analyzed in this work, these characteristics were considered to distinguish main channels and branches. From these considerations, cases dubiously suggesting two or more main channels for the same lightning were not considered to not make the mistake of treating an intense and large branch as the main channel or reject a main channel as if it was a branch.

The procedures of distinction and recognition between channels and branches can interfere in the results. For example, the choice of the leader channel or return stroke main channel as a lightning channel can generate discrepancies among the results obtained from these different choices for lightning channel. However, as still there is not a specific definition of lightning channel, there is no sense in saying that by choosing one or another channel definition for a problem to be treated, it is incurring some error. A systematic, cautious and thorough verification of discrepancies due to different definitions of lightning channel is suggested for a future work.

The data consisted of 78 filmed lightning organized in the categories presented at the introduction of this work. For each movie, the frame containing the lightning channel was extracted and converted into photos with size of 704×480 pixels. For this extraction the software *Virtual Dub* was used. The photo was loaded into the software *Grapher* from which the channel was digitized. In the digitization the software defined the height of the photo as the *Y* (vertical) axis and the width as the *X* (horizontal) axis. Scanning of the channel was carefully carried out point to point from bottom to top, using the zoom feature offered by the software, when needed. When a branch was reached in a point of the channel, the scanning along the channel was paused at that point, and from that point a scanning along the branch was started. After the scanning along the branch, the scanning of the channel continued from the point where it was paused. The coordinates (*X*, *Y*) of each point scanned along the channel and branches were digitized into American Standard Code for Information Interchange (ASCII) and stored in a worksheet containing two columns, the first for *X* coordinate and the second for Y coordinate. The graphic of these coordinates shows the image of lightning, as can be seen in Fig. 2(a and b). Linear Interpolation between pairs of coordinates was done until the number of points in the digitized structure was equal to 2^{14}, minimizing lacunarity due to scanning method. These ASCII data are computational and mathematical representation of natural lightning captured in two-dimensional images. These data were used as input into a computer program to calculate the multifractal spectrum.

For generation of data representative of simulated three-dimensional lightning, a third coordinate (*Z*) was added to each (*X*, *Y*) coordinate of data representative of two-dimensional natural lightning, generating a (*X*, *Y*, *Z*) coordinate which represents a point in the channel of the simulated three-dimensional lightning. These data in ASCII format were stored in worksheets with three columns. The *Z* coordinate was computed by weighted average of the coordinates (*X*, *Y*) representative of a point in the channel of two-dimensional natural lightning. The values of the statistical weights for the coordinates *X* and *Y* were selected from a uniform distribution in the interval between 0 and 1, using Monte Carlo simulation. By visual inspection, when the simulation generated a three-dimensional lightning with shapes of lightning channels very different from those usually observed for a real lightning, the simulated lightning was rejected. Of course, this procedure takes into account the expertise obtained during visual observations of natural lightning.

This care enables to take the assumption of the three-

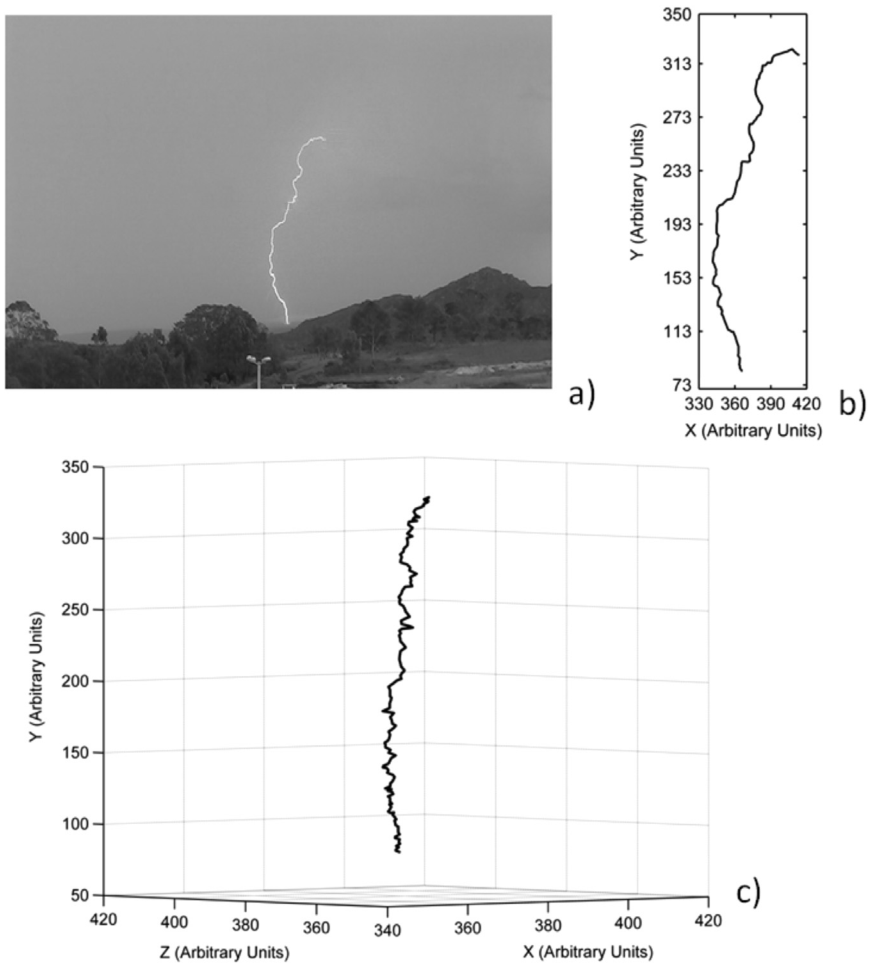

Fig. 2. Graphical representation of a filmed lightning: a) Natural lightning included in ground lightning category and non-branched ground lightning category; b) Graphical representation of lightning shown in a) through digitized coordinates (*X*, *Y*) of its channel; c) Perspective view of a three-dimensional lightning simulated from data presented in b).

dimensional simulated lightning (or simple three-dimensional lightning) as a supposed natural lightning in three dimensions, and the two-dimensional image from which it was generated as its projection on a plane. Fig. 2c illustrates the data representative of three-dimensional lightning.

2.2. *Multifractal theory and singularity spectrum computation*

This paper gives a brief overview of the fundamentals of Multifractal theory. Those interested in further approach are encouraged to see Vicsek (1992).

A fractal curve is an object that has a fractal dimension (D) between 1 and 2, while a fractal surface has a fractal dimension (D) between 2 and 3 (Lupò et al., 2000). Thus, a fractal object occupies more space than a standard euclidian object. For objects which are not fractals, their fractal dimensions coincide with their topological dimensions (d=1 for curves, d=2 for surfaces).

A method to determine the fractal dimension of an object with maximal linear length *L* and embedded in a *d*-dimensional Euclidian space, is the *Box-Count*. In this method the object is covered with (hyper)-boxes of length $\varepsilon \le L$. The volume of the (hyper)-boxes is ε^d, where $d \ge D$. The number (*N*) of (hyper)-boxes covering the object obeys a power-law and is:

$$N(\varepsilon) \approx \varepsilon^D \tag{1}$$

The fractal dimension can be calculated as follows:

$$D = \lim_{\varepsilon \to 0} \frac{\log[N(\varepsilon)]}{\log[\varepsilon]} \tag{2}$$

It can happen that some objects of studies, also termed *measures*, have their geometric structure or statistical properties distributed not only in one, but in several subsets, each with its own fractal dimension. It has then a multifractal object. In trust, multifractals are formed by interwoven of fractals subsets with different scaling exponents (Souza and Rostirolla, 2011). In this case, multifractals can be described in terms of their Generalized Dimensions D_q or in terms of their Multifractal (or Singularity) Spectra $f(\alpha)$. To compute D_q or $f(\alpha)$ of a measure, one can cover the support of the measure with (hyper)-boxes of length ε and define

$$P_i(\varepsilon) = N_i(\varepsilon)/N \tag{3}$$

as the probability of the *i*th box, where $N_i(\varepsilon)$ is the number of points of the measure inside the *i*th box and *N* is the total number of points in the measure. This probability scales as

$$P_i(\varepsilon) \approx \varepsilon^{\alpha_i} \tag{4}$$

Where α_i is singularity strength or Lipschitz–Hölder exponents (Chhabra and Jensen, 1989; Souza and Rostirolla, 2011). The number of boxes in which $P_i(\varepsilon)$ has singularity strength equals α also scales as

$$N_\alpha(\varepsilon) \approx \varepsilon^{f(\alpha)} \tag{5}$$

To determine the generalized dimensions D_q a partition function is defined as

$$Z_q(\varepsilon) = \sum_i^{n(\varepsilon)} [P_i(\varepsilon)]^q \tag{6}$$

Where $n(\varepsilon)$ is the number of boxes in the scale ε covering the measure and q is a parameter ($q \in R$) called moment of order q.

According to Chhabra and Jensen (1989) and Souza and Rostirolla (2011) the generalized dimension D_q is

$$D_q = \frac{1}{q-1} \lim_{\varepsilon \to 0} \frac{\log(Z_q(\varepsilon))}{\log(\varepsilon)} = \frac{1}{q-1} \tau(q) \tag{7}$$

Where $\tau(q)$ is the mass exponent, and, D_q and $f(\alpha)$ are related by a Legendre transform. However, according to Chhabra and Jensen (1989) the method of Legendre transform for determining $f(\alpha)$ from $\tau(q)$ presents some problems, as example, possibility of missing some discontinuities ("phase transitions") on $\tau(q)$ or $f(\alpha)$ curve. Their alternative procedure, the *direct evaluation* solves this problem and f and α can be evaluated as functions of q as

$$f(q) = \lim_{\varepsilon \to 0} \frac{\sum_i \mu_i(q, \varepsilon) \log[\mu_i(q, \varepsilon)]}{\log(\varepsilon)} \tag{8}$$

$$\alpha(q) = \lim_{\varepsilon \to 0} \frac{\sum_i \mu_i(q, \varepsilon) \log[P_i(\varepsilon)]}{\log(\varepsilon)} \tag{9}$$

Where

$$\mu_i(q, \varepsilon) = \frac{[P_i(\varepsilon)]^q}{Z_q(\varepsilon)} \tag{10}$$

One of the problems of using Box-Count method is that the counts of boxes and computation of their probabilities (Eq. (3)) can cost a long time and large amount of memory. However, Souza and Rostirolla (2011) developed an algorithm which is able to perform fast counts, followed by $f(\alpha)$ computation via Legendre transform.

In order to evaluate the performance of their algorithms, Chhabra and Jensen (1989) and Souza and Rostirolla (2011) have used two-scale Cantor measures for which theoretical multifractal spectra were obtained and used to verify the reliability of the numerical multifractal spectra provided by the algorithms. The numerical results provided by the algorithms of Chhabra and Jensen (1989) and Souza and Rostirolla (2011) showed good approximation to the theoretical results. In this work, we chose to combine the efficiency of box counts of Souza and Rostirolla (2011) and the procedure of direct evaluation of Chhabra and Jensen (1989), which does not have the disadvantages of Legendre transform. The implemented algorithm was evaluated with the same data used by Souza and Rostirolla (2011) and generated good results. Thus, it was used to compute multifractal spectra of data in this work.

3. Results and discussion

According to Rodriguez-Iturbe and Rinaldo (1997) one important feature of the multifractal spectrum concerns its shape. The shape of a multifractal spectrum can be symmetric or asymmetric. A symmetric multifractal has a symmetrical convex curve shape (∩ shape), and it is found when the measure is self-similar and its multifractal spectrum is a continuous and twice differentiable function (Halsey et al., 1986). However, some measures found in nature do not share these features and their multifractal spectra have asymmetrical curve shape or even, a hook-like shape. An asymmetrical multifractal spectrum is an indication of a self-affine measure or departure of self-similarity. In this work, all categories of lightning exhibited only asymmetric spectra.

Another issue to be considered is the limitation of numerical procedures to extract multifractal spectrum from data. According to Rodriguez-Iturbe and Rinaldo (1997) and Veneziano et al. (1995), when the actual spectrum of the measure is not continuous and convex, as usually assumed, numerical algorithms yield an envelope or convex hull of the true $f(\alpha)$ and the consequences are: (i) spurious points may be introduced; (ii) "interior" points (spectrum points out of the envelope) are not captured. These consequences result in apparent multifractal spectra which make measures which are not multifractal at all appear multifractal. Thus, care must be taken in order not to commit an erroneous interpretation of the numerical results. However, according to Rodriguez-Iturbe and Rinaldo (1997), three important features of the multifractal spectrum should be considered genuine: the minimum and maximum values of α and the maximum value of $f(\alpha)$.

In order to check whether spurious points were introduced by the estimation algorithm used in this work, we recalculated the multifractal spectra using the procedure of exclusion of portions of the two end sides of the data (Veneziano et al., 1995) and compared them with the multifractal spectra of full data. In most of these observations, changes in spectra of both full and cut data were exhibited, indicating occurrences of spurious points. The region of the multifractal spectra corresponding to the negative values of moments (q) suffered more severe changes, while the region corresponding to non-negative values of moments (q) suffered negligible changes, regardless the category of lightning. This suggests that possibly the actual spectrum of lightning channel is not continuous and the values of $f(\alpha)$ for non-negative moments (q) are more reliable, despite the limitation of the numerical procedure. Because of this limitation, we have been careful to interpret the results just as suggestive or not of the occurrence of multifractal character and restrict ourselves only to the extraction of information considered as genuine. A more conclusive determination of the multifractal character of lightning channel is an issue that requires further study to be conducted in the future.

The following sections present the results of multifractal analysis of lightning in different categories. For such analysis, the average multifractal spectrum was computed from individual spectra of lightning in a determined category of lightning. In order to verify the reliability of the computed average values and compare average multifractal spectra of different categories of lightning, the standard error of the mean was used.

3.1. *Multifractal analysis of two-dimensional and three-dimensional lightning*

For simplification of the simulation in the verification of the influence of embedding dimension in the multifractal spectrum, just lightning without branches and showing a single channel during its evolution were selected. This selection resulted in an amount of 36 two-dimensional lightning, from which three-dimensional lightning where obtained.

Fig. 3 shows the average multifractal spectra of two-dimensional and three-dimensional lightning.

The $f(\alpha)$ curves in Fig. 3 were computed for the range of $q \in [-10, 10]$ with step equal to 0.5. The points of maximum at the $f(\alpha)$ curves represent the values of α and $f(\alpha)$ related to $q = 0$. The points to the left correspond to $q > 0$ and the points to the right correspond to $q < 0$.

Note in Fig. 3a that both spectra seem to be different, once they are shifted along the horizontal and vertical axes.

To investigate the vertical shift between two-dimensional and three-dimensional spectra, for each value of q, the discrepancies between two average values of $f(\alpha)$ of both spectra and

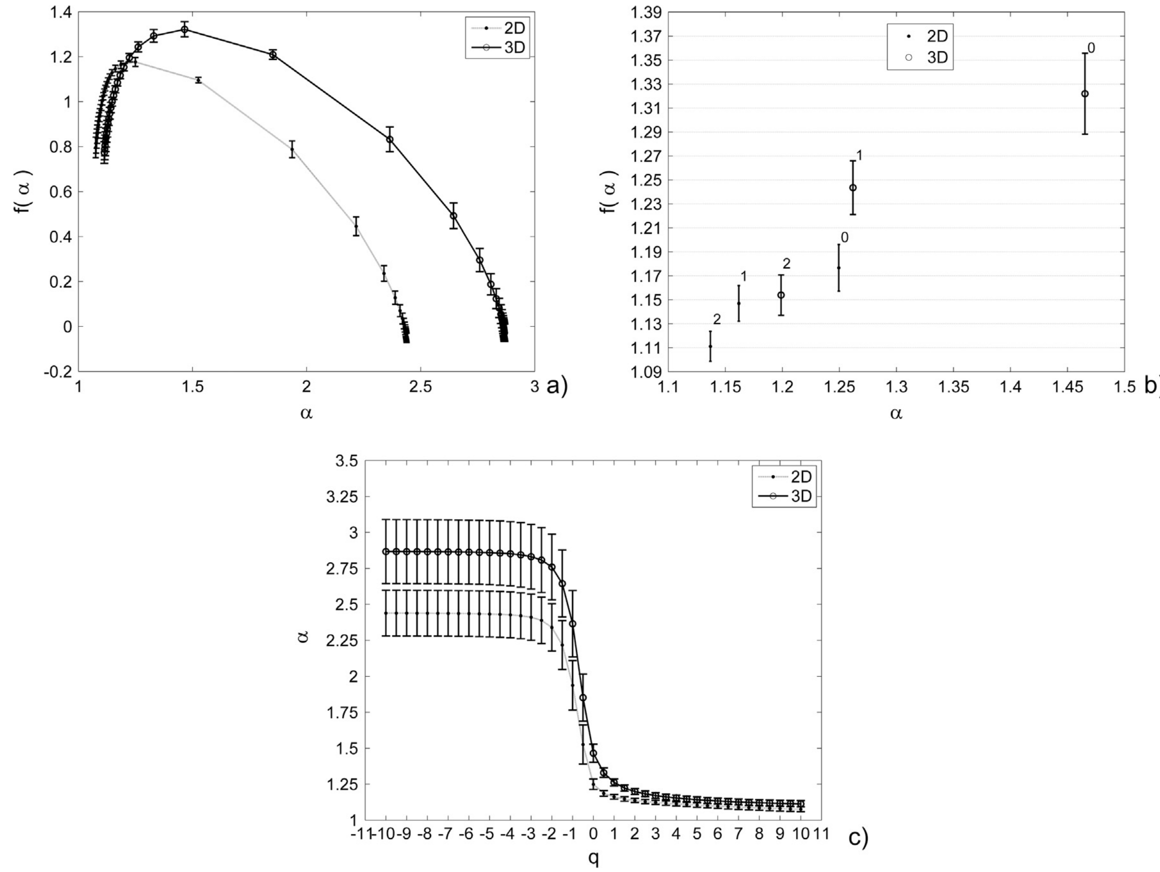

Fig. 3. Multifractal analysis of two-dimensional lightning (2D) and three-dimensional lightning (3D): a) Multifractal spectra; b) Multifractal spectra for moments 0, 1, and 2; c) Singularity strength versus moment.

correspondent to the same value of q were analyzed through their standard error. Thus, in Fig. 3a the error bars represent the variability of the average values of $f(\alpha)$. A careful inspection of the error bars in Fig. 3a was carried out and for about 86% of the moments (q) the error bars overlapped, indicating that these differences are statistically insignificant, except for the values of q equal to 0, 1 and 2, which error bars did not overlap, as shown in Fig. 3b, indicating that at these values of q the average values of $f(\alpha)$ of both spectra are really different.

For the investigation of the horizontal shift, a similar procedure was performed taking into account average values of α at both spectra and for a determined value of q. Fig. 3c shows values of average α in function of values of q. A careful inspection of the error bars in Fig. 3c was carried out and no error bars overlapped, indicating that the differences between two average values of α of both spectra and for the same value of q are statistically significant for any value of q.

The differences present in some average values of $f(\alpha)$ and in all average values of α confirm that multifractal spectra of two-dimensional lightning and three-dimensional lightning are different. This result suggests that multifractal spectrum of a real lightning is different of its estimate obtained from two dimensional images.

Differently of monofractals, which spectrum is a point, Fig. 3a shows both multifractal spectra wide and curved. These wide and curved spectra suggest multifractality (Gou et al. 2007, 2009). Fig. 3a suggests that the multifractal character of the three-dimensional lightning is preserved even when the multifractal spectrum is obtained from two-dimensional lightning. Thus, this result suggests that the use of two-dimensional data of real lightning embedded in three dimensions does not impede a qualitative study, such as the observation of the multifractal character of lightning channels.

However, a quantitative study of real lightning from multifractal spectrum obtained from two-dimensional lightning seems to be impaired due to the differences associated to shifts at both spectra. According to Gou et al. (2007, 2009) a wider spectrum indicates a higher degree of multifractality. Thus, Fig. 3a suggests a higher degree of multifractality for three-dimensional lightning which is related mainly to the significant horizontal shift. This result also suggests an underestimating of the multifractal spectrum of real lightning when estimating it from two dimensional images.

In fact, the results seem to indicate that two-dimensional data of lightning channel embedded in three dimensions do not to reveal all the true complexity of the structure of lightning channels, since, the analysis from two-dimensional data would be omitting information about the structure.

As already discussed, the use of two-dimensional data to estimate the multifractal spectrum of real lightning, generate significant discrepancies in all average values of α in the spectrum and only in the average values of $f(\alpha)$ corresponding to values of q equal to 0, 1 and 2. Even discrepant, the average values of $f(\alpha)$ for $q = 0, 1, 2$ have interesting interpretations and further discussion worth.

For $q = 0$ it can be shown that

$$f(\alpha) = \lim_{\varepsilon \to 0} \frac{\log\left(\sum_i^{n(\varepsilon)} P_i(\varepsilon) \right)}{\log(\varepsilon)} = D_0 \tag{11}$$

where D_0 is the fractal dimension of the measure. For $q = 1$, it can be shown that

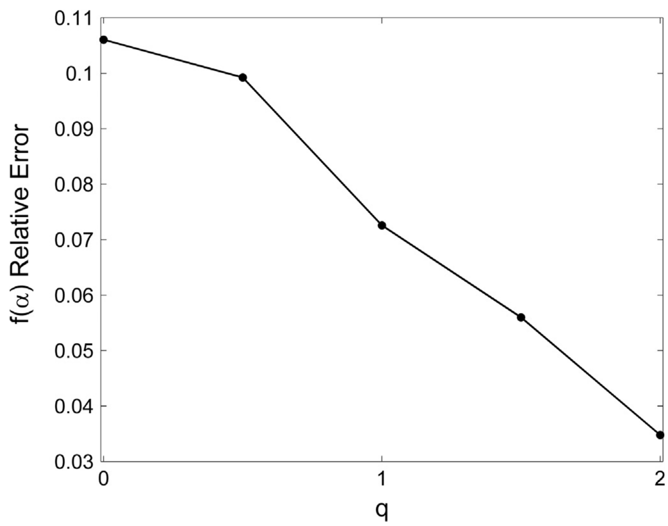

Fig. 4. Relative errors between average values of $f(\alpha)$ of two-dimensional and three-dimensional lightning for moments q equal to 1, 2 and 3.

$$f(\alpha) = \lim_{\varepsilon \to 0} \frac{\sum_i^{n(\varepsilon)} P_i(\varepsilon) \log[P_i(\varepsilon)]}{\log(\varepsilon)} = D_1 \tag{12}$$

where D_1 is the information dimension of the measure. For $q = 2$, it can be shown that

$$f(\alpha) = \lim_{\varepsilon \to 0} \frac{\log\left[\sum_i^{n(\varepsilon)} [P_i(\varepsilon)]^2\right]}{\log(\varepsilon)} = D_2 \tag{13}$$

where D_2 is the correlation dimension of the measure.

The relative errors of $f(\alpha)$ and α are presented in Figs. 4 and in 5, respectively. They were computed assuming the values obtained from three-dimensional lightning as "true" and the values obtained from two-dimensional lightning as their estimates. Fig. 4 shows the relative errors between average values of $f(\alpha)$ in the spectrum of two-dimensional lightning and average values of $f(\alpha)$ in the spectrum of three-dimensional lightning, for values of q which differences are statistically significant. These average values of $f(\alpha)$ are shown in Table 1 as generalized dimensions (D_0, D_1 and D_2).

In our work, we found D_0 for two-dimensional lightning about 10.61% less than D_0 for three-dimensional lightning, a result in conformity with the results obtained by Sañudo et al. (1995) and Amarashinge and Sonnadara (2008). In addition, we also observed

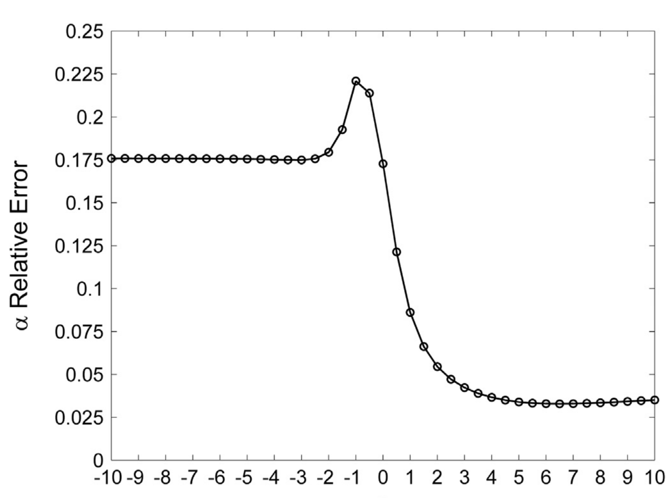

Fig. 5. Relative errors between average values of α of two-dimensional (2D) and three-dimensional (3D) lightning in function of moments q.

Table 1
Average values of generalized dimensions of two-dimensional (2D) and three-dimensional (3D) lightning.

	D_0	D_1	D_2
2D Lightning	1.18 ± 0.02	1.15 ± 0.01	1.11 ± 0.01
3D Lightning	1.32 ± 0.03	1.24 ± 0.02	1.15 ± 0.02

a reduction of about 7.26% for D_1 and about 3.48% for D_2, for two-dimensional lightning channels, compared with the respective values for three-dimensional lightning channels. Thus, we suggest these values as pattern parameters of deviation, when estimating the generalized dimensions D_0, D_1 and D_2, of real lightning from their two-dimensional images.

The values presented in Table 1 were extracted from $f(\alpha)$ spectra in Fig. 3a. One can see in Fig. 3b and Table 1a value of D_0 for three-dimensional lightning larger than D_0 for two-dimensional lightning. This is explained as follows: in the simulation, the insertion of Z coordinate increases the tortuosity (and possible insert more branches and nodes) which contributes to a larger occupation of space. As D_0 is a measurement of the space occupied by a measure in the embedding space, a higher result for D_0 is expected for three-dimensional lightning.

Fig. 3b and Table 1 show a value of D_1 for three-dimensional lightning larger than D_1 for two-dimensional lightning. This is explained based on the meaning of entropy and information. The Eq. (12) is related to the definition of statistical entropy ($S = \sum_i^k P_i \log[P_i]$). This entropy is related to the possibilities of a physical system occupy different states in a sample with k possible states. In fact, in the case of the description of physical systems, the diversity of possible states for a physical system corresponds to the disorder of the system and also to the amount of inherent information when making a complete description of the physical system. The greater the number of possible states in a physical system, the greater its disorder and its inherent amount of information.

In this paper, the image of lightning is embedded in a two-dimensional or three-dimensional Cartesian coordinate system. Each point in this system can be thought as a site with possible information, "on" if it contains part of the lightning channel or "off" if it does not contain part of the lightning channel. In the simulation, at the instant of a random generation of a point in the channel, any pixel of the coordinate system can be selected. In this sense, in a three-dimensional coordinate system the possibilities of configurations for lightning channel is greater than in the two-dimensional case, since that at the generation of a point of the channel, the number of combinations between three coordinates (x, y, z) is greater than the number of combinations of two coordinates (x, y).

In analogy to statistical mechanics, one can say that configurations of channels are states and lightning is the system. A specific lightning channel configuration is a state among all possible states permitted by the coordinate system.

A more tortuous and branched channel, has larger number of sites with "*on*" information. In fact, the greater this number, the greater the number of possible configurations of channel permitted by the coordinate system, resulting in more disorder of the system and more intrinsic information in the system. As in the three-dimensional simulation, the insertion of the Z coordinate generates more tortuosity and branches, resulting in greater number of sites "*on*" and a larger information dimension is expected for the case of three-dimensional channels.

All this also contributes to the fact that the actual lightning has additional nodes and different length ratios regarding their two-dimensional projections. The expected results are different spectra for both categories in question.

Fig. 3b and Table 1 show that three-dimensional lightning channels on average exhibit slightly larger correlation.

Fig. 5 shows relative errors between average values of α in both spectra as a function of the values of q. Note that the estimate of α of three-dimensional obtained from two-dimensional data leads to considerable relative errors for $q < 0$, presenting values between 17% and 23%. For $q > 0$ is less affected, presenting a relative error equal to 17.3% when $q = 0$ and smallest values for $q > 0$. Thus, the use of two-dimensional lightning for determination of α values of real lightning is less accurate for $q < 0$.

Given what has been discussed so far in this work, next sections present multifractal analyses for different categories. The analyses were performed from two-dimensional lightning and care was taken to restrict the analyses to the qualitative observation of the occurrence of mono or multifractality and at most the quantitative determination of the values of D_0, D_1 and D_2.

3.2. Multifractal analysis of branched and non-branched ground lightning

For this analysis images of 36 non-branched ground lightning and 14 branched ground lightning were selected. All these lightning had a single channel during occurrence. Fig. 6a shows the spectra for these categories in study.

Analyses of the standard errors and their error bars of the average values of $f(\alpha)$ and of the average values of α were performed. For the case of average values of $f(\alpha)$ no overlap of error bars occurred and the discrepancies between the average values of $f(\alpha)$ of both spectra are significant, indicating different spectra for non-branched ground and branched ground categories. Regarding the average values of α, the error bar analysis indicated significant discrepancies only for non-negative values of moments (Fig. 6c), also confirming that the spectra of non-branched ground and branched ground categories are different.

Both multifractal spectra in Fig. 6a are wide and curved and thus suggest multifractality for these categories of lightning. Fig. 6b and Table 2 show that average values of the generalized dimensions D_0, D_1 and D_2 are larger in branched ground lightning when compared with the non-branched ground lightning. This result is explained as follows: the branches requires more space and add more intrinsic information to channels, increasing complexity in channel structure and leading to larger values of dimensions. In this work we found that the average generalized dimensions D_0, D_1 and D_2 of non-branched ground lightning are respectively, about 9.92%, 11.54% and 12.59% smaller than the average generalized dimensions D_0, D_1 and D_2 of branched ground lightning.

3.3. Multifractal analysis of ground and cloud lightning

For this analysis the ground lightning category considered lightning with a single channel and no branches, lightning with a single channel and branches and lightning with multiple channels, which struck the ground. These data consisted of 71 lightning channels. For cloud lightning category, we selected lightning with similar possibilities for characteristics of channels, except that they did not strike the ground. These data consisted of 23 lightning. Fig. 7a and Table 3 show the average multifractal spectra and

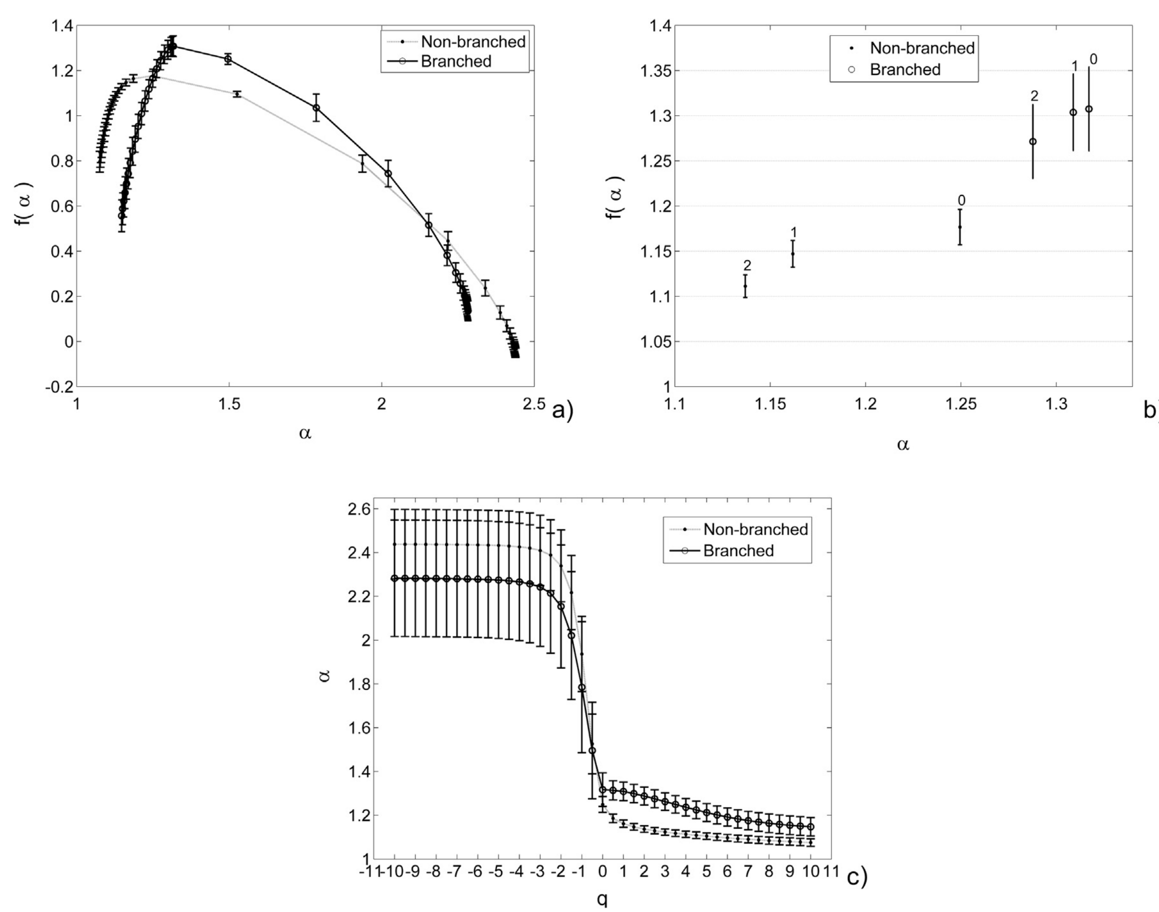

Fig. 6. Multifractal analysis of non-branched ground and branched ground lightning: a) Multifractal spectra; b) Multifractal spectra for moments 0, 1, and 2; c) Singularity strength versus moment.

Table 2
Average values of generalized dimensions of non-branched ground and branched ground lightning.

	D_0	D_1	D_2
Non-branched	1.18 ± 0.02	1.15 ± 0.01	1.11 ± 0.01
Branched	1.31 ± 0.05	1.30 ± 0.04	1.27 ± 0.04

generalized dimensions of both categories.

An analysis of the standard error was performed and significant discrepancies among average values of $f(\alpha)$ of both spectra were found only for $q \geq -1$, except for values 3 and 4. For the case of average values of α (Fig. 7c), significant discrepancies between both spectra were found only for values of $q \in [0, 3]$. These results confirm that both spectra are really different.

Fig. 7a shows wide and curved spectra, suggesting that both categories of lightning have strong multifractality. The multifractal spectrum of cloud lightning category is wider than the multifractal spectrum of ground lightning category, suggesting a larger degree of multifractality for cloud lightning. Furthermore, Fig. 7b and Table 3 show that the average generalized dimensions (D_0, D_1, D_2) of cloud lightning category are larger than the average generalized dimensions of ground lightning, suggesting that cloud lightning channels have more complexity than ground lightning channels. It is possible that this result is due to the fact that a cloud lightning usually has more branches.

In this work we found that the average generalized dimensions (D_0, D_1, D_2) of ground lightning are respectively, about 6.92%, 6.30% and 4.17% smaller than the average generalized dimensions (D_0, D_1, D_2) of cloud lightning.

Table 3
Average values of generalized dimensions of ground and cloud lightning.

	D_0	D_1	D_2
Ground	1.21 ± 0.02	1.19 ± 0.02	1.15 ± 0.01
Cloud	1.30 ± 0.03	1.27 ± 0.02	1.20 ± 0.03

3.4. Multifractal analysis of single and multiple lightning

For this analysis 21 single lightning and 33 multiple lightning with only one channel were selected. Fig. 8a shows the multifractal spectra for both categories of lightning.

An analysis of the standard error was performed and significant discrepancies among average values of $f(\alpha)$ of both spectra were found only for $|q| > 4$. For the case of average values of α (Fig. 8c) no significant discrepancy between both spectra was found.

These results suggest that the differences between both spectra are mainly due to discrepancies in some average values of $f(\alpha)$, while the values of singularity strength in both categories are statistically the same, suggesting the same types of singularities in both spectra. Thus, one cannot tell which spectrum is the widest. However, both spectra are curved and wide suggesting multifractality for both categories of lightning. Fig. 8b and Table 4 present values of generalized dimensions D_0, D_1 and D_2 for both categories and show no significant discrepancy. All these results contribute in the suggestion that the fact of lightning being single or multiple does not appear to interfere in the complexity of channel structure of both categories of lightning. Further, both categories have the same average fractal, information and correlation dimensions.

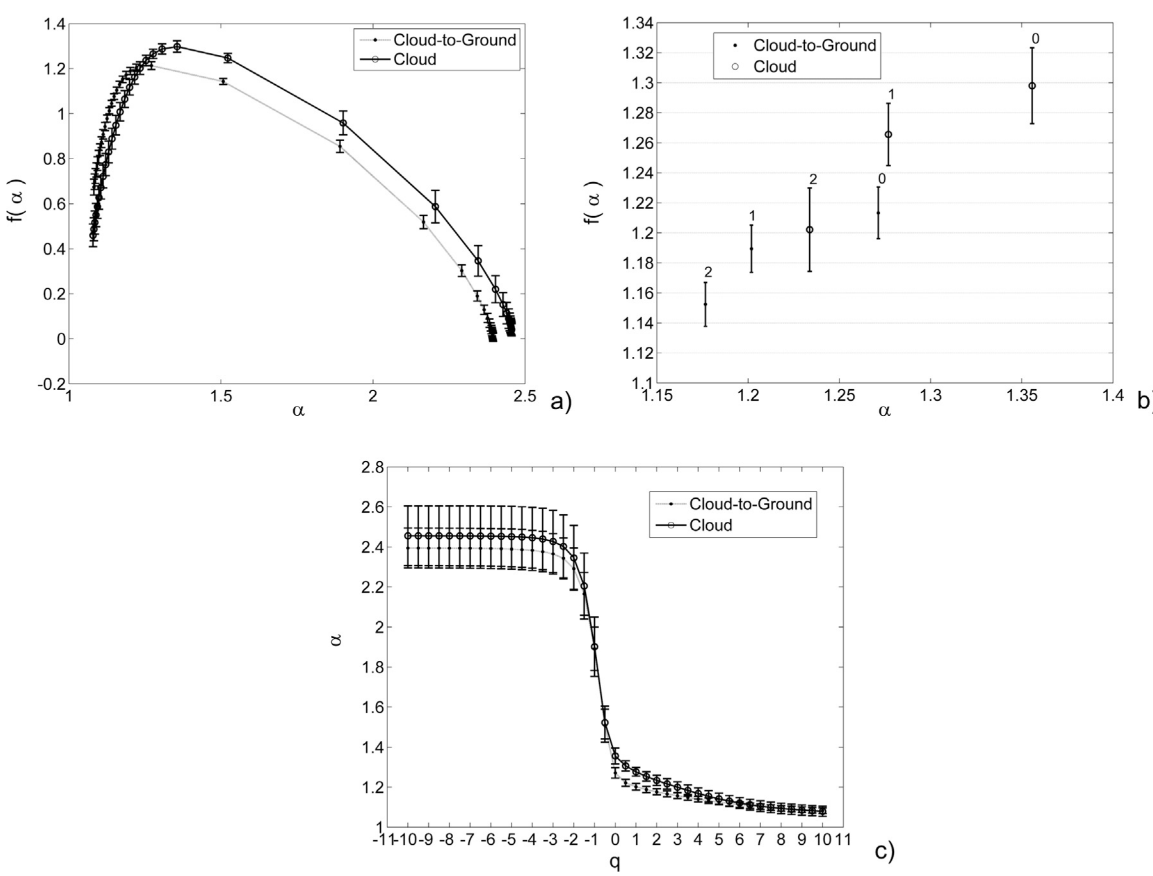

Fig. 7. Multifractal analysis of cloud-to-ground and cloud lightning: a) Multifractal spectra; b) Multifractal spectra for moments 0, 1, and 2; c) Singularity strength versus moment.

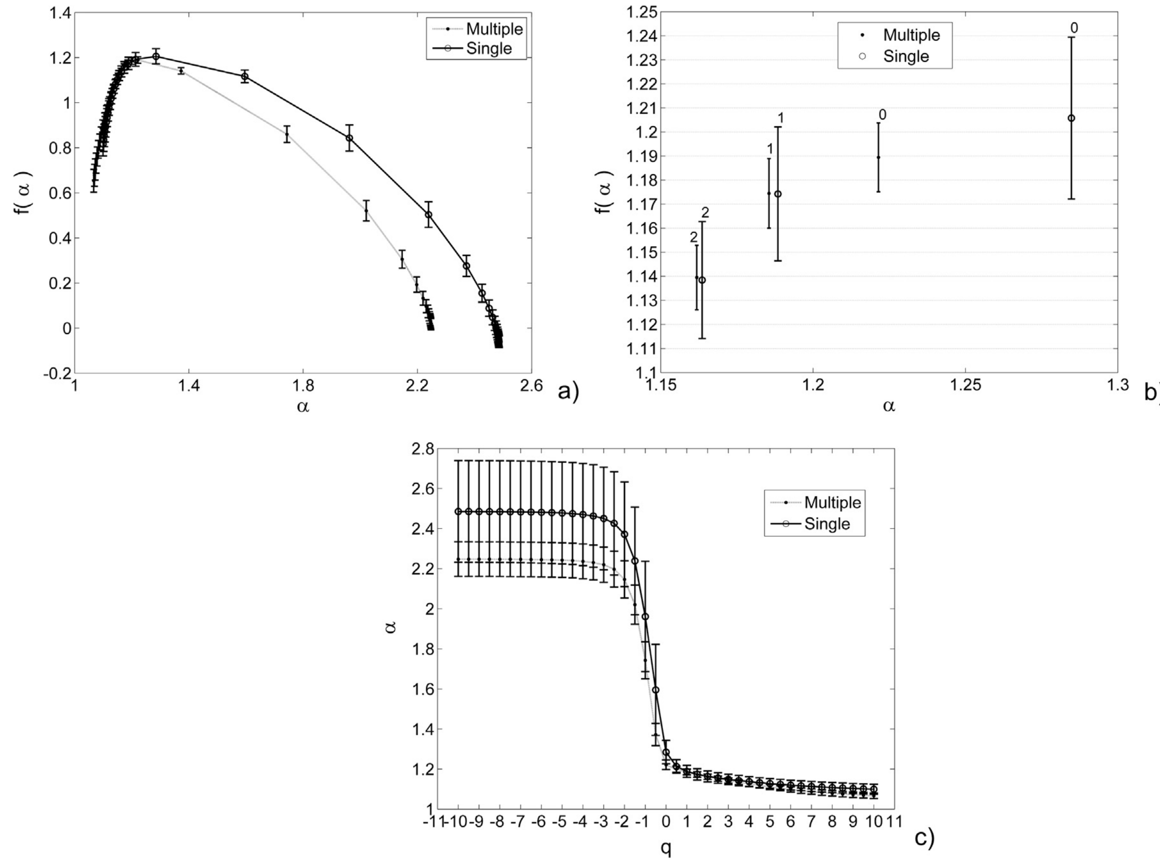

Fig. 8. Multifractal analysis of single and multiple lightning: a) Multifractal spectra; b) Multifractal spectra for moments 0, 1, and 2; c) Singularity strength versus moment.

Table 4
Average values of generalized dimensions for multiple and single lightning.

	D_0	D_1	D_2
Multiple	1.19 ± 0.01	1.17 ± 0.01	1.14 ± 0.01
Single	1.21 ± 0.03	1.17 ± 0.03	1.13 ± 0.02

4. Conclusions

To the best of our knowledge, this is the first multifractal analysis of channels of lightning classified in different categories. In this study, about 78 lightning were organized in the different categories of lightning for the following comparisons: two-dimensional lightning versus three-dimensional lightning, non-branched ground lightning versus branched ground lightning; ground lightning versus cloud lightning; single lightning versus multiple lightning.

The results suggest that the multifractal spectra of natural lightning in three dimensions, when estimated from their two-dimensional images are underestimated in their values. This effect is much more pronounced for negative values of the generalized moments (q) than for positive values. However, some information of natural lightning in three dimensions is preserved in the multifractal spectrum estimated from two-dimensional images. The first information is qualitative and it is about the mono or multifractality of lightning channels. The results suggest that a lightning in three dimensions is multifractal and this information is also suggested when observing the spectrum obtained from its two-dimensional images. The second information that can be leveraged is quantitative. It is about the values of $f(\alpha)$ corresponding to D_0, D_1 and D_2. Values of D_0 of two-dimensional projections of three-dimensional simulated lightning have been determined in previous works with values about 10–13% smaller than the values of three-dimensional simulated lightning. In our work, this reduction is about 10%, and is a result in conformity with the literature. The results show that, when using two-dimensional images to perform multifractal analysis of lightning in three dimensions, the analysis should be restricted to the qualitative identification of mono or multifractal character of lightning, or at most, the estimate of fractal dimensions with a deviation between 10% and 13% of their real values. We also observed the possibilities of estimating information and correlation dimensions of a three-dimensional lightning from its two dimensional images. In this case reductions about 7% in the values of information dimension and about 3% in the correlation dimension were found.

Thus, when using two-dimensional images to perform multifractal analysis of lightning in the various categories, we were careful to restrict ourselves to the verification of mono or multifractality of lightning channels and the quantization of fractal dimensions. The results suggest multifractality for all categories of lightning in our work. It was found that average values of fractal dimensions can be grouped according to categories of lightning.

When comparing branched and non-branched ground lightning, the results suggest that branches require more occupation and insert more intrinsic information in the channel, increasing the complexity of channels and leading to largest average values of fractal dimensions. Average values of fractal dimension, information dimension and correlation dimension of non-branched ground lightning are respectively about 9%, 12% and 13%, smaller than these dimensions for branched ground lightning.

Cloud lightning exhibited average values of fractal dimensions larger than the average values of ground lightning. In this work, it was observed that cloud lightning were usually more branched and tortuous than ground lightning, leading to largest average values of fractal dimensions. Average values of fractal dimension, information dimension and correlation dimension for ground lightning are respectively about 7%, 16% and 4%, smaller than these dimensions for cloud lightning.

The results suggest that the complexity of the structure of lightning channels does not depend on the fact of lightning being single or multiple. Furthermore, they suggest that both categories have similar complexities in their channel structures, leading to the same average fractal, information and correlation dimensions.

Acknowledgements

The authors would like to thank the Fundação de Amparo à Pesquisa do Estado de Minas Gerais (FAPEMIG - process APQ 01513-08) for the financial support for this study, Souza, J. de for the fruitful discussions and for providing the algorithm that has been adapted and used at this work and Araújo, C.O. for the help in data manipulation.

References

Amarasinghe, D., Sonnadara, U., 2008. Fractal characteristics of simulated electrical discharges. J. Natl. Sci. Found. Sri Lanka 36 (2), 137–143.
Antunes, L.S., Saraiva, A.C.V., Pinto Jr., O., Alves, J., Campos, L.Z.S., Luz, E.S.A.M., Medeiros, C., Buzato, T.S., 2015. Day-to-day differences in the characterization of lightning observed by multiple high-speed cameras. Electr. Power Syst. Res 118, 93–100.
Chhabra, A., Jensen, R.V., 1989. Direct determination of the $f(\alpha)$ singularity spectrum. Phys. Rev. Lett. 62 (12), 1327–1330.
Esa, M.R.M., Ahmad, M.R., Cooray, V., 2014. Wavelet analysis of the first electric field pulse of lightning flashes in Sweden. Atmos. Res. 138 (1), 235–267.
Flache, D., Rakov, V.A., Heidler, F., Zischank, W., Thottappillil, R., 2008. Initial-stage pulses in upward lightning: leader/return stroke versus M-component mode of charge transfer to ground. Geophys. Res. Lett. 35, L13812. http://dx.doi.org/10.1029/2008GL034148.
Gou, X., Zhang, Y., Dong, W., Qie, X., 2007. Wavelet-based multifractal analysis of the radiation field of first return stroke in cloud-to-ground discharge. Chinese J. Geophys. 50 (1), 104–109.
Gou, X., Chen, M., Zhang, Y., Dong, W., Qie, X., 2009. Wavelet multiresolution based multifractal analysis of electric fields by lightning return strokes. Atmos. Res. 91, 410–415.
Halsey, T.C., Jensen, M.H., Kadanoff, L.P., Procaccia, I., Shraiman, B.I., 1986. Fractal measures and their singularities: The characterization of strange sets. Phys. Rev. A 33 (2), 1141–1151.
Hayakawa, M., Iudin, D.I., Mareev, E.A., Trakhtengerts, V.Y., 2007. Cellular automaton modeling of mesospheric optical emissions: sprites. Phys. Plasmas. 14 (042902). http://dx.doi.org/10.1063/1.2721079.
Hayakawa, M., Iudin, D.I., Trakhtengerts, V.Y., 2008. Modeling of thundercloud VHF/UHF radiation on the lightning preliminary breakdown stage. J. Atmos. Sol. Terr. Phys. 70, 1660–1668.
Horner, F., Bradley, P.A., 1964. Spectra of atmospherics from near lightning. J. Atmos. Terr. Phys. 26, 1155–1166.
Idone, V.P., Orville, R.E., 1988. Channel tortuosity variation in Florida triggered lightning. Geophy. Res. Lett. 15 (7), 645–648.
Idone, V.P., 1995. Microscale tortuosity and its variation as observed in triggered lightning channel. J. Geophys. Res. 100 (D11), 22,943–22,956.
Iudin, D.I., Trakhtengerts, V.Y., Hayakawa, M., 2003. Fractal dynamics of electric discharges in a thundercloud. Phys. Rev. E 68 (016601). http://dx.doi.org/10.1103/PhysRevE.68.016601.
Kawasaki, Z., Matsuura, K., 2000. Does a lightning channel show a fractal? Appl. Energy 67, 147–158.
Kitagawa, N., Brook, M., 1960. A comparison of intracloud and cloud-to-ground lightning discharges. J. Geophys. Res. 65, 1189–1201.
Le Vine, D.M., Meneghini, R., 1978. Simulation of radiation from lightning return strokes: The effects of tortuosity. Radio Sci. 13 (5), 801–809.
Le Vine, D.M., 1987. Review of measurements of the RF spectrum of radiation from lightning. Meteorol. Atmos. Phys. 37 (3), 195–204.
Li, Q., Li, K., Chen, X., 2013. Research on lightning electromagnetic fields associated with first and subsequent return strokes based on Laplace wavelet. J. Atmos. Sol. Terr. Phys. 93, 1–10.
Lupò, G., Petrarca, C., Tucci, V., Vitelli, M., 2000. EM fields associated with lightning channels: on the effect of tortuosity and branching. IEEE Trans. Electromagn. Compat. 42 (4), 394–404.
Miranda, F.J., Pinto Jr., O., Saba, M.M.F., 2003. A study of the time interval between return strokes and K-changes of negative cloud-to-ground lightning flashes in Brazil. J. Atmos. Sol. Terr. Phys. 65, 293–297.
Miranda, F.J., 2008. Wavelet analysis of lightning return stroke. J. Atmos. Sol. Terr. Phys. 70, 1401–1407.
Niemeyer, L., Pietronero, L., Wiesmann, H.J., 1984. Fractal dimension of dielectric breakdown. Phys. Rev. Lett. 52 (12), 1033–1036.
Perera, M.D.N., Sonnadara, D.U.J., 2013. Fractal nature of simulated lightning channels. Sri Lanka J. Phys. 13 (2), 09–25.
Rodriguez-Iturbe, I., Rinaldo, A., 1997. Fractal River Basins – Chance and Self-Organization, 1st ed. Cambridge University Press, Cambridge, United Kingdom, p. 547.
Saba, M.M.F., Ballarotti, M.G., Pinto Jr., O., Miranda, F.J., Naccarato, K.P., 2004. Simultaneous electric field and high-speed video observations of lightning. In: Proceedings of the International Conference on Grounding and Earthing & 1st International Conference on Lightning Physics and effects GROUND'2004 and 1st LPE., 2004, Belo Horizonte. pp. 38–42.
Sañudo, J., Gómez, J.B., Castaño, F., Pacheco, A.F., 1995. Fractal dimension of lightning discharge. Nonlinear Process. Geophys. 2, 101–106.
Schafer, J.P., Goodall, W.M., 1939. Peak field strengths of atmospherics due to local thunderstorms at 150 megacycles. Proc. IRE 27, 202–207.
Schonland, B.F.J., Malan, D.J., Collens, H., 1935. Progressive lightning, 2. Proc. R. Soc. Lond., Ser. A 152, 595–625.
Schonland, B.F.J., Malan, D.J., Collens, H., 1938. Progressive lightning, 6. Proc. R. Soc. Lond., Ser. A 168, 455–469.
Serhan, G.I., Uman, M.A., Childers, D.G., Lin, Y.T., 1980. The RF spectra of first and subsequent lightning return strokes in the 1-200 km range. Radio Sci. 15, 1089–1094.
Sharma, S.R., Cooray, V., Fernando, M., Miranda, F.J., 2011. Temporal features of different lightning events revealed from wavelet transform. J. Atmos. Sol. Terr. Phys. 73, 507–515.
Souza, J., Rostirolla, S.P., 2011. A fast MATLAB program to estimate the multifractal spectrum of multidimensional data: application to fractures. Comput. Geosci. 37, 241–249.
Thottappillil, R., Rakov, V.A., Uman, M.A., 1990. K and M changes in close lightning ground flashes in Florida. J. Geophys. Res. 95, 18,631–18,640.
Tsonis, A.A., 1991. A fractal study of dielectric breakdown in the atmosphere. In: Schertzer, D., Lovejoy, S. (Eds.), Scalong, Fractals and Non Variability in Geophysics. Kluwer, The Netherlands, pp. 167–174.
Valdivia, J.A., Milikh, G., Papadopoulos, K., 1997. Red sprites: lightning as a fractal antenna. Geophy. Res. Lett. 24 (24), 3169–3172.
Valdivia, J.A., Milikh, G.M., Papadopoulos, K., 1998. Model of red sprites due to intracloud fractal lightning discharges. Radio Sci. 33 (6), 1655–1668.
Valine, W.C., Krider, E.P., 2002. Statistics and characteristics of cloud-to-ground lightning with multiple ground contacts. J. Geophys. Res. 107 (D20), 4441. http://dx.doi.org/10.1029/2001JD001360.
Vecchi, G., Labate, D., Canavero, F., 1994. Fractal approach to lighting radiation on a tortuous channel. Radio Sci. 29 (4), 691–704.
Veneziano, D., Moglen, G.E., Bras, R.L., 1995. Multifractal analysis: Pitfalls of standard procedures and alternatives. Phys. Rev. E 52 (2), 1387–1398.
Vicsek, T., 1992. Fractal growth phenomena, 2nd ed. World Scientific, Singapore, New Jersey, p. 488.
Winn, W.P., Aldridge, T.V., Moore, C.B., 1973. Video-tape recordings of lightning. J. Geophys. Res. 78 (21), 4515–4519.
Willett, J.C., Le Vine, D.M., Idone, V.P., 1995. Lightning-channel morphology revealed by return-stroke radiation field waveforms. J. Geophys. Res. 100 (D2), 2727–2738.